Int. J. Heat Mass Transfer. Vol. 35, No. 10, pp. 2491–2499, 1992
Printed in Great Britain

0017–9310/92 \$5.00+0.00

Effects of anisotropic scattering on radiative heat transfer using the P_1-approximation

FENGSHAN LIU
Department of Fuel and Energy, University of Leeds, Leeds LS2 9JT, U.K.

and

E. S. GARBETT and J. SWITHENBANK
Department of Mechanical and Process Engineering, University of Sheffield, Sheffield S1 3JD, U.K.

(*Received* 21 *June* 1991 *and in final form* 25 *October* 1991)

Abstract—The P_1-approximation method, along with the δ-Eddington phase function approximation, was used to study radiative heat transfer in absorbing–emitting–scattering media. It is established that the asymmetry factor of the scattering phase function (g) plays an important role in radiative transfer. The concepts of the effective scattering coefficient and effective optical dimension are suggested to be used in the study of radiative heat transfer. In highly forward scattering media ($g > 0.8$), the effect of scattering on radiative heat transfer can be neglected. In a two-dimensional square enclosure, the results of radiative heat flux obtained by the P_1-approximation, using an improved boundary condition, are in good agreement with the numerically exact results of Kim and Lee (Effect of anisotropic scattering on radiative heat transfer in two-dimensional rectangular enclosures, *Int. J. Heat Mass Transfer* **31**, 1711–1721 (1988)).

1. INTRODUCTION

INCREASING research efforts have been paid to treating anisotropic scattering because of the importance of this effect in atmospheric radiation transfer, pulverised coal-fired combustion systems, and many other areas. However, radiative transfer in an absorbing, emitting, and scattering medium is governed by a complex integro-differential equation which is difficult and computationally expensive to solve in multi-dimensional geometries.

Recently, the radiative transfer equation in multi-dimensional absorbing, emitting and scattering media has been solved numerically by various approximate schemes. Menguc and Viskanta used the P_1- and the P_3-approximations along with the δ-Eddington phase function approximation to study the effect of anisotropic scattering on radiative heat transfer [1, 2]. In their version of the P_1- and the P_3-approximations, the values of the two phase function parameters of the δ-Eddington approximation are required to perform the calculation. Their numerical results show a dramatic effect of the scattering phase function and single scattering albedo on radiative heat flux. Later work of Truelove [3] employing the S_4-approximation of the discrete ordinates method indicated that the numerical results of Menguc and Viskanta are unreliable. More recently, Kim and Lee incorporated a complex Mie scattering phase function into the S_{14}-approximation to predict radiative heat transfer in a two-dimensional rectangular enclosure containing grey, absorbing, emitting, and scattering media [4]. Their results can be regarded as numerically exact.

Over the years, attempts have been made to study the effect of anisotropic scattering from fly-ash particles on radiative heat transfer in pulverised coal-fired furnaces. For example, the P_3-approximation results of Menguc and Viskanta [5] show that anisotropic scattering has a significant effect on the radiative heat flux distribution at the wall of a large scale pulverised coal-fired furnace and conclude that neither the isotropic nor the non-scattering assumptions can be made. Nevertheless, the quantitative effect has not been clearly established. It is, therefore, of great value to ascertain the magnitude of this effect in order to make any realistic modelling assumptions for simplifying the calculation of the in-scattering term.

In the prediction of pulverised coal combustion where the radiative transfer equation is solved simultaneously with other transport equations governing the conservation of mass, momentum, species and energy, economic measures must be taken into account even with the loss of some accuracy. A simple, accurate, and computationally efficient radiation model is highly desirable to be incorporated into a general prediction procedure. The P_1-approximation offers the advantages of simplicity, high computational efficiency and capability of treating anisotropic scattering. Moreover, it has been demonstrated that in absorbing–emitting media the accuracy of the P_1-approximation can be improved significantly when an optimised boundary condition is employed [6].

NOMENCLATURE

g asymmetry factor of scattering phase function
I radiation intensity [W m^{-2} sr^{-1}]
I_0 zeroth moment of radiation intensity [W m^{-2}]
I_i first-order moments of radiation intensity [W m^{-2}]
I_{ij} second-order moments of radiation intensity [W m^{-2}]
I_b black-body radiation intensity [W m^{-2} sr^{-1}]
l_i direction cosines: ξ if $i = 1$; η if $i = 2$; μ if $i = 3$
L characteristic dimension [m]
Q non-dimensional net radiative heat flux
$\mathbf{r}$ spatial location vector
s distance measured along the direction of radiation propagation [m]
S volumetric heat generation rate [kW m^{-3}]
T temperature [K]
x, y, z Cartesian coordinates [m].

Greek symbols
ε emissivity
θ polar angle [rad]
κ_a absorption coefficient [m^{-1}]
κ_e extinction coefficient, $\kappa_a+\kappa_s$ [m^{-1}]
κ_e' effective extinction coefficient [m^{-1}]
κ_s scattering coefficient [m^{-1}]
κ_s' effective scattering coefficient [m^{-1}]
ξ, η, μ direction cosines
τ_0 optical dimension, $\kappa_e L$
ϕ azimuthal angle [rad]
Φ scattering phase function
Ψ scattering angle [rad]
ω single scattering albedo, κ_s/κ_e
Ω solid angle [sr].

Subscripts
a anisotropic scattering
i isotropic scattering
w wall.

Therefore, it is desirable to investigate the accuracy of the P_1-approximation in scattering media in order to extend its capacity of application.

In this work, the accuracy of the P_1-approximation in absorbing, emitting and scattering media when using an improved boundary condition is studied by comparing the results of this work with those of Kim and Lee [4]. The δ-Eddington phase function approximation [7, 8], which represents a general Mie scattering phase function in a simple yet accurate fashion, is incorporated into the P_1-approximation. It is found in this work that the effect of scattering can be evaluated without knowing the δ-Eddington phase function parameters as long as the asymmetry factor of the original phase function is available. Effects of the phase function and scattering coefficient on radiative heat transfer are also studied using the present P_1-approximation.

2. FORMULATION

The radiative transfer equation in an absorbing, emitting and scattering grey medium in local thermodynamic equilibrium can be written as

$$\frac{dI}{ds} = -\kappa_a I - \kappa_s I + \kappa_a I_b + \kappa_s \frac{1}{4\pi}\int_{\Omega'=4\pi} \Phi(\Psi) I(\Omega')\,d\Omega'. \tag{1}$$

In the Cartesian coordinates system, the derivative with respect to the distance s is given as

$$\frac{d}{ds} = \frac{\partial}{\partial x}\xi + \frac{\partial}{\partial y}\eta + \frac{\partial}{\partial z}\mu \tag{2}$$

where ξ, η and μ are direction cosines defined as

$$\xi = \sin\theta\cos\phi \quad \eta = \sin\theta\sin\phi \quad \mu = \cos\theta. \tag{3}$$

In the spherical harmonics method, the moments of the intensity are used as dependent variables in calculating the intensity and radiative heat flux. The moments of intensity are defined as the integrals of intensity over the entire solid angle after first multiplying by the appropriate direction cosines such that

$$I_0(\mathbf{r}) = \int_0^{2\pi}\int_0^{\pi} I(\mathbf{r},\theta,\phi)\sin\theta\,d\theta\,d\phi \tag{4}$$

$$I_i(\mathbf{r}) = \int_0^{2\pi}\int_0^{\pi} l_i I(\mathbf{r},\theta,\phi)\sin\theta\,d\theta\,d\phi \tag{5}$$

$$I_{ij}(\mathbf{r}) = \int_0^{2\pi}\int_0^{\pi} l_i l_j I(\mathbf{r},\theta,\phi)\sin\theta\,d\theta\,d\phi \tag{6}$$

where l_i and l_j are the direction cosines and each of them is either ξ, η or μ.

In the P_1-approximation, the radiation intensity is expanded in terms of its moments as [1]

$$I(\mathbf{r},\theta,\phi) = \frac{1}{4\pi}[I_0 + 3(I_1\xi + I_2\eta + I_3\mu)]. \tag{7}$$

The closure conditions for the P_1-approximation are

$$I_{ij} = \tfrac{1}{3} I_0 \delta_{ij}. \tag{8}$$

Before going further to formulate the governing equations of P_1-approximation, the scattering phase function must be considered in order to work out the expression for the in-scattering term. The exact phase function can be expanded in a series of Legendre polynomials such that

$$\Phi(\Psi) = \sum_{n=0}^{\infty} a_n P_n(\cos \Psi), \tag{9}$$

where a_n is the angular distribution coefficient which can be calculated using the orthogonality property of the Legendre polynomials. Multiplying equation (9) by $P_n(\cos \Psi)$ and integrating results in

$$a_n = \frac{2n+1}{4\pi} \int_{\Omega=4\pi} \Phi(\Psi) P_n(\cos \Psi)\, \mathrm{d}\Omega. \tag{10}$$

In the simple P_1-approximation, the use of such a complicated phase function becomes unnecessary. It is believed that the simple yet accurate δ-Eddington phase function approximation is appropriate for the P_1-approximation which takes the form

$$\Phi_a(\Psi) = 2f\delta(1-\cos\Psi) + (1-f)(1+3g'\cos\Psi). \tag{11}$$

The parameter g', which is the asymmetry factor of the truncated phase function, is determined by assuming that the approximate phase function $\Phi_a(\Psi)$ has the same asymmetry factor g as the exact phase function. The asymmetry factor is an important parameter defined as

$$g = \frac{1}{4\pi} \int_{\Omega=4\pi} \Phi(\Psi) \cos\Psi\, \mathrm{d}\Omega. \tag{12}$$

It has been stressed that g is the fundamental phase function similarity parameter [9]. The asymmetry factor represents the amount of radiation scattered in the forward direction. For example, $g = 1$, 0 and -1 correspond to complete forward scattering, isotropic scattering, and complete backward scattering, respectively. Although the asymmetry factor can be calculated through the Mie theory [8], the method is very complicated and not practical for engineering applications. Fortunately, the asymmetry factor g can be determined experimentally. Boothroyd *et al.* have presented some data for the asymmetry factor of fly-ash [10]. Both the experimental results of Boothroyd *et al.* [10] and the calculation of Goodwin and Mitchner [11] indicate that fly-ash particles have an asymmetry factor about 0.8.

The asymmetry factor of the δ-Eddington phase function can be obtained by using the definition of the asymmetry factor, equation (12), such that

$$\frac{1}{4\pi} \int_{\Omega=4\pi} \Phi_a(\Psi) \cos\Psi\, \mathrm{d}\Omega = f + (1-f)g'. \tag{13}$$

Requiring the asymmetry factor of the δ-Eddington approximation to be equal to the asymmetry factor g of the original phase function results in

$$g = f + g' - fg'. \tag{14}$$

The parameter g' is then given as

$$g' = \frac{g-f}{1-f}. \tag{15}$$

In fact, the δ-Eddington phase function parameters f and g' can be related to a_n coefficients (a_1 and a_2) of equation (10) by replacing $\Phi_a(\Psi)$ for $\Phi(\Psi)$ [1, 8]. However, it has been observed that the use of f and g' determined in this way yields negative phase function values at some scattering angles [8]. It will be shown that in the P_1-approximation the determination of f and g' is not necessary if the asymmetry factor of the original phase function g is known.

The expression for the in-scattering term in equation (1) can be obtained by employing the δ-Eddington phase function approximation given in equation (11) and written as

$$\kappa_s \frac{1}{4\pi} \int_{\Omega'=4\pi} \Phi_a(\Psi) I(\Omega')\, \mathrm{d}\Omega' = \kappa_s f I + \kappa_s \frac{1}{4\pi}(1-f)[I_0 + 3g'(I_1\xi + I_2\eta + I_3\mu)]. \tag{16}$$

After performing some standard derivations, see Menguc and Vistanka [1], the governing equation of the P_1-approximation in the Cartesian coordinates system can be obtained and written as

$$\frac{\partial}{\partial x}\left[\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial z}\right] = \kappa_a(I_0 - 4\pi I_b). \tag{17}$$

If the volumetric heat generation rate S of the medium is specified, the governing equation of the P_1-approximation then takes the form

$$\frac{\partial}{\partial x}\left[\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial y}\right] + \frac{\partial}{\partial z}\left[\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial z}\right] = S. \tag{18}$$

The net radiative heat fluxes, I_i, are related to I_0 through

$$I_1 = -\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial x} \tag{19}$$

$$I_2 = -\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial y} \tag{20}$$

$$I_3 = -\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial z} \tag{21}$$

where the parameter κ_e' is given as

$$\kappa_e' = \kappa_a + (1 - f - g' + fg')\kappa_s. \tag{22}$$

A close look at these equations reveals that what is needed is a linear combination of f and g', i.e. $f + g' - fg'$, rather than individual f or g'. Fortunately, this linear combination is just equal to the asymmetry factor of the original phase function g as given in equation (14). Therefore, it is not necessary to calculate the individual values of f and g' when the asymmetry factor g is available and this is often the case for most scattering problems. Equation (22) can be written in terms of g as

$$\kappa_e' = \kappa_a + (1-g)\kappa_s. \tag{23}$$

In this study, the new physical quantity κ_e' is termed the *effective extinction coefficient* and suggested to be used in the study of radiative heat transfer. The prime used for κ_a' is to distinguish it from the conventional *extinction coefficient* κ_e defined as

$$\kappa_e = \kappa_a + \kappa_s. \tag{24}$$

The effective extinction coefficient is related to the extinction coefficient through

$$\kappa_e' = (1-\omega g)\kappa_e. \tag{25}$$

Another quantity κ_s' is defined as

$$\kappa_s' = (1-g)\kappa_s. \tag{26}$$

As the effect of scattering depends on both the scattering coefficient and the phase function of the medium, we call κ_s' the *effective scattering coefficient* as this quantity takes into account both factors. It is also suggested in this work that the *effective optical dimension* of a radiation system defined as $\kappa_s' L$ should be introduced in the study of radiative heat transfer.

In a two-dimensional axisymmetric coordinate system, the governing equations of the P_1-approximation have been obtained by Liu [12] and are given as

$$I_1 = -\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial r} \tag{27}$$

$$I_3 = -\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial z} \tag{28}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{1}{3\kappa_e'}r\frac{\partial I_0}{\partial r}\right] + \frac{\partial}{\partial z}\left[\frac{1}{3\kappa_e'}\frac{\partial I_0}{\partial z}\right] = \kappa_a(I_0 - 4\pi I_b). \tag{29}$$

In this study, the generalised boundary condition of Liu *et al.* [6] is used to obtain the numerical results of the P_1-approximation. Under the assumptions that all surface walls are grey and diffusively emitting and reflecting, the generalised boundary condition contains an arbitrary constant and takes the form [6, 12]

$$I_0 \pm \frac{3k+2(1-\varepsilon_w)}{\varepsilon_w} I_i = 4\pi I_b(T_w) \tag{30}$$

where $k = (n+1)/(n+2)$, n is an arbitrary positive integer, and $\pm$ corresponds to the surfaces in the negative and positive directions, respectively. The physical significance of n and the effect of n on the results of the P_1-approximation have been discussed and studied in ref. [6]. k may be explained as a certain modification to the wall emissivity.

Equation (23) indicates that for complete forward scattering where g is equal to unity, the effect of scattering vanishes. This is true simply because in this case the increase in radiation intensity due to in-scattering is completely compensated by its decrease due to out-scattering, see equation (1). Grosshandler and Monteiro used this result as an approximation to study the absorptivity of pulversied coal particles experimentally [13].

It is worth noting that a scaling law can be readily derived from the governing equations of the P_1-approximation. A comprehensive discussion of scaling laws which reduce anisotropic scattering problems to isotropic scattering ones has been presented by McKellar and Box [14]. Equation (23) and equations (17)–(21) indicate that the characteristic physical parameters which determine the solution of the zeroth moment and first-order moments are

$$\kappa_a \quad \text{and} \quad (1-g)\kappa_s.$$

It should be pointed out that the corresponding boundary condition, equation (30), does not contribute any new parameters to the problem. Therefore, it is possible to reduce an anisotropic problem to an isotropic problem through the scaling

$$\kappa_a|_i = \kappa_a|_a \tag{31}$$

$$[(1-g)\kappa_s]|_i = [(1-g)\kappa_s]|_a. \tag{32}$$

Bear in mind that for isotropic scattering $g = 0$. Equation (32) can be written in a clearer form

$$\kappa_s|_i = (1-g)\kappa_s|_a. \tag{33}$$

This scaling, formed by equations (31) and (33), is equivalent to that established by Lee and Buckius [15] based on the P_1-approximation for one-dimensional problems. In ref. [15], Lee and Buckius show numerically that this scaling yields more accurate results than other scaling schemes for one-dimensional radiative transfer problems. More recently, the accuracy of this scaling has been investigated by Kim and Lee [16] employing the S_{14}-approximation for two-dimensional problems. Their results show that this scaling is very accurate for diffuse incidence and isothermal emission problems.

3. RESULTS AND DISCUSSION

The P_1-approximation was applied to predict radiative heat transfer in a two-dimensional square enclosure since for this problem the S_{14}-approximation solutions of Kim and Lee [4], which can serve as numerically exact solutions, have been presented in the literature.

Numerical results of the P_1-approximation reported in this paper were obtained by using the elliptic equation successive-overrelaxation (SOR) iterative technique. It is assumed that the convergence is achieved when the maximum percentage error of the zeroth moment of radiation intensity is less than 0.001%. All the numerical results of the P_1-approximation were obtained by employing a 20 × 20 uniform grid scheme and on the AMDAHL computer at the University of Leeds, U.K. In this work, four representative scattering phase functions are considered, highly forward scattering phase functions F1 and F2, isotropic scattering, and backward scattering function B2. The symmetry factors of these phase functions are 0.85, 0.67, 0.0 and −0.4, respectively. Details of phase functions F1, F2, and B2 can be found in ref. [4]. One of the advantages of the simple P_1-approximation is its computational efficiency. For the physical parameters studied in this work, the longest CPU time is 60 s which occurs when the square has a very low wall emissivity ($\varepsilon_w = 0.1$) and contains a pure highly forward scattering medium, Fig. 3. It was found that the CPU time per run depends strongly on the wall emissivity and the effective optical dimension. The higher the wall emissivity and the effective optical dimension, the lower the CPU time required. For black square enclosures and pure scattering media, a 20 s CPU time is required for highly forward scattering problems. For isotropic and backward scattering media, however, the CPU time is less than 5 s. Based on the previous study [6] and some numerical sensitivity studies of the effect on n on the P_1-approximation results for the problems considered, the integer n in the boundary condition is assumed to be 10 in the present study.

3.1. *Boundary incidence problems*

The first case studied is a boundary incidence problem where the bottom wall of the square enclosure has a unit emissive power E_{bw} but all other walls and the medium are kept cold. The length of the side wall of the enclosure is 1 m. It is believed that the effect of anisotropy on radiative heat transfer is most significant in this non-symmetric situation. The predictions of the P_1-approximation are compared with the results of Kim and Lee [4] for different phase functions, scattering albedos, wall emissivities, and optical dimensions.

Figure 1 shows the effect of phase function (asymmetry factor) on the centreline net radiative heat flux in the y direction of a black square enclosure for pure scattering media. The P_1-approximation results are in good agreement with the numerically exact solutions except for the highly forward scattering case where g is equal to 0.85. This is indeed expected because in this case the medium is nearly non-scattering (and also non-absorbing) and the radiation intensity is highly dependent on direction. For isotropic and backward scattering, the distribution of radiation intensity is less directional dependent and, therefore, the P_1-approximation is accurate. The P_1-approximation underpredicts the centreline net radiative heat flux near the cold wall. The effect of the asymmetry factor on the centreline radiative heat flux is significant for this non-symmetric heat input problem. In fact, the change in asymmetry factor alters the effective optical dimension of the square enclosure. The P_1-approximation

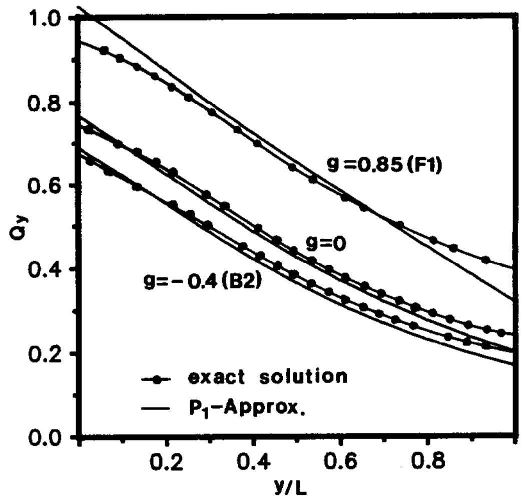

FIG. 1. Centreline net radiative heat flux in the y direction: $\varepsilon_w = 1.0$, $L = 1.0$ m, $\kappa_a = 0.0$, $\kappa_s = 1.0$ m^{-1}.

results are not in good agreement with the exact results for highly forward scattering because the effective optical dimensions are very small. In the extreme situation of complete forward scattering, the effective optical dimension is zero and the medium transfers the most radiative heat.

In forward scattering media (phase function F2), the effect of scattering albedo on the centreline net radiative heat flux in the y direction is shown in Fig. 2. The results obtained by the P_1-approximation are in good agreement with the S_{14}-approximation results except in the region near the cold wall where the P_1-approximation again underpredicts the radiative heat fluxes. The results show that the centreline net radiative heat flux decreases with increasing the scattering albedo. This effect of the scattering albedo can be

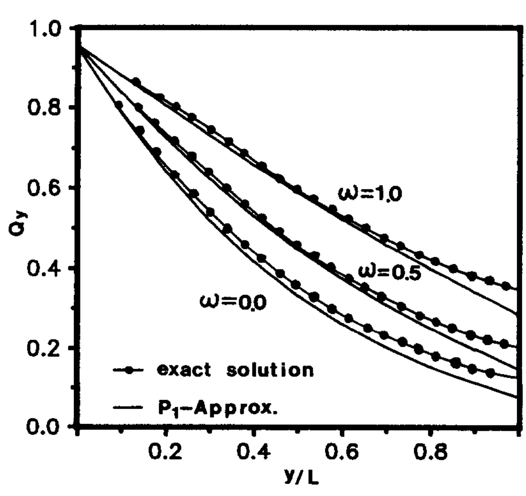

FIG. 2. Effect of scattering albedo on the centreline net radiative heat flux in the y direction: $\varepsilon_w = 1.0$, $L = 1.0$ m, $g = 0.67$ (F2), $\kappa_e = 1.0$ m^{-1}.

explained in terms of the effective optical dimension. For forward scattering ($g > 0$), the increase of the scattering albedo causes the decrease of the effective optical dimension, see equation (25), and, therefore, increases the centreline net radiative heat flux.

Figure 3 shows the centreline net radiative heat fluxes in the y direction in a grey square enclosure of unity optical dimension containing a pure forward scattering medium (phase function F2) for different wall emissivities. It can be seen that the effect of wall emissivity is significant because the radiative heat emitted from the hot wall is proportional to the wall emissivity. The agreement between the P_1-approximation results and the exact results is excellent especially for small wall emissivities.

The effect of optical dimension on the centreline net heat flux in the y direction is studied in a black square enclosure containing a pure forward scattering medium (phase function F2). The results are compared with the exact results in Fig. 4. In general, the P_1-approximation predictions are in better agreement with the exact solutions with increasing optical dimension. In fact, the increase of optical dimension in this pure scattering medium has the same effect as decreasing the asymmetry factor, see Fig. 1, because they both increase the effective optical dimension of the enclosure.

Figure 5 shows the net radiative heat fluxes at the hot wall surface for different phase functions. In the case of highly forward scattering, the P_1-approximation yields the worst results in comparison with the exact results, about 10% higher than the exact solution and unrealistically higher than unity. This unphysical behaviour of the P_1-approximation has been discussed in detail by Ratzel and Howell [17]. However, when the asymmetry factors are small ($g = 0$ and -0.4), the agreement between the P_1-approximation predictions and the exact results is very good. This is simply because the radiation intensity is less directional dependent with a decreasing asymmetry factor. The net heat flux on the hot wall surface decreases when the asymmetry factor increases. This effect is due to the increase of the incident heat flux to the hot wall when the backward scattering becomes significant.

The P_1-approximation has also been used to examine the relative importance of absorption and scattering to radiative heat transfer for this boundary incidence problem. To this end, the medium is assumed to be absorbing and isotropic scattering. The results are shown in Figs. 6 and 7. It can be seen that for this boundary incidence problem both absorption and scattering coefficients have a significant effect on the centreline net radiative heat flux in the y direction.

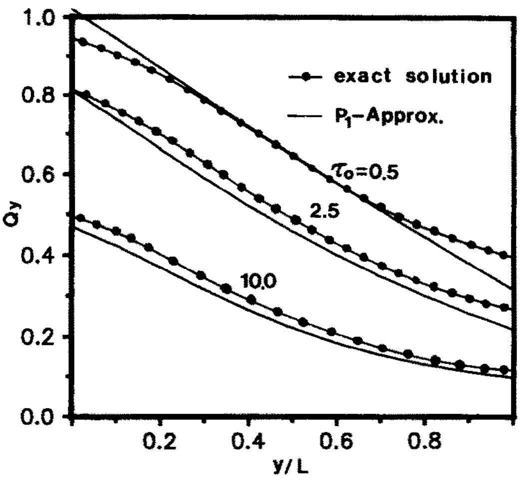

FIG. 4. Effect of optical dimension on the centreline net radiative heat flux in the y direction: $\varepsilon_w = 1.0$, $L = 1.0$ m, $g = 0.67$ (F2), $\omega = 1.0$.

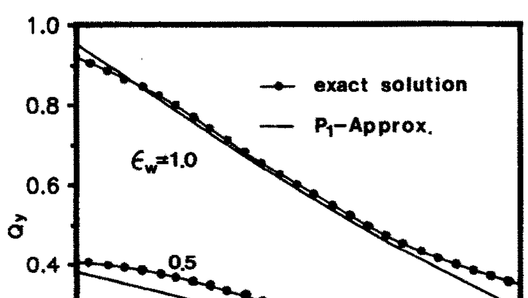

FIG. 3. Effect of wall emissivity on the centreline net radiative heat flux in the y direction: $\omega = 1.0$, $L = 1.0$ m, $g = 0.67$ (F2), $\kappa_e = 1.0\ m^{-1}$.

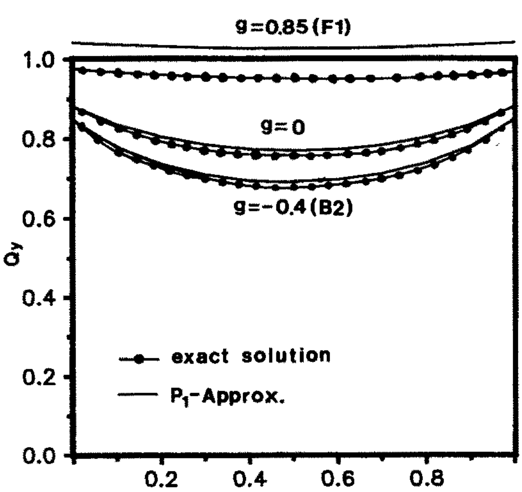

FIG. 5. Effect of anisotropy on the hot surface net radiative heat flux: $\varepsilon_w = 1.0$, $L = 1.0$ m, $\omega = 1.0$, $\kappa_e = 1.0\ m^{-1}$.

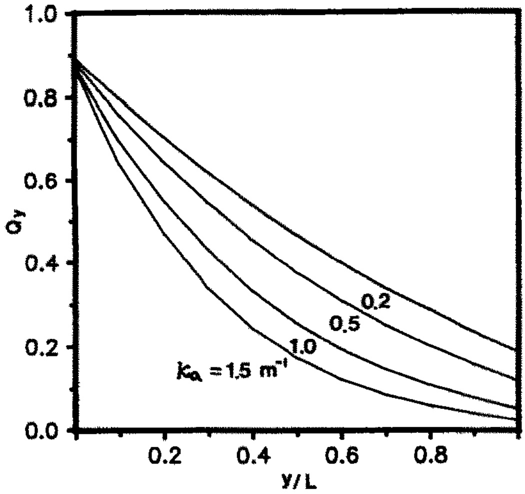

FIG. 6. Effect of absorption coefficient on the centreline net radiative heat flux in the y direction: $\varepsilon_w = 1.0$, $L = 1.0$ m, $g = 0.0$, $\kappa_s = 0.5$ m^{-1}.

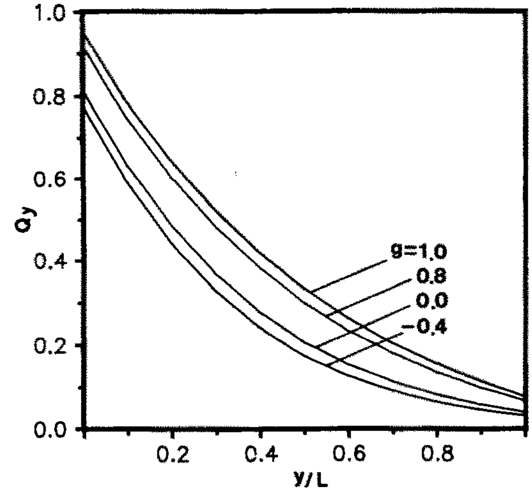

FIG. 8. Effect of asymmetry factor on the centreline net radiative heat flux in the y direction: $\varepsilon_w = 1.0$, $L = 1.0$ m, $\kappa_a = 1.0$ m^{-1}, $\kappa_s = 1.0$ m^{-1}.

It should be noted that the absorption coefficient has negligible effect on the radiative heat flux on the hot wall surface. Figure 8 shows the effect of the asymmetry factor on the centreline radiative heat flux when the medium is absorbing and scattering. When scattering is highly forward, the scattering effect can be neglected by treating the medium as non-scattering without causing great error. The practical conclusion drawn from these results is that the effect of scattering from fly-ash particles can be neglected with confidence. Therefore, the problem of predicting radiative heat transfer in pulverised coal-fired furnaces can be greatly simplified. However, the absorption and emission of radiation by fly-ash particles must be taken into account.

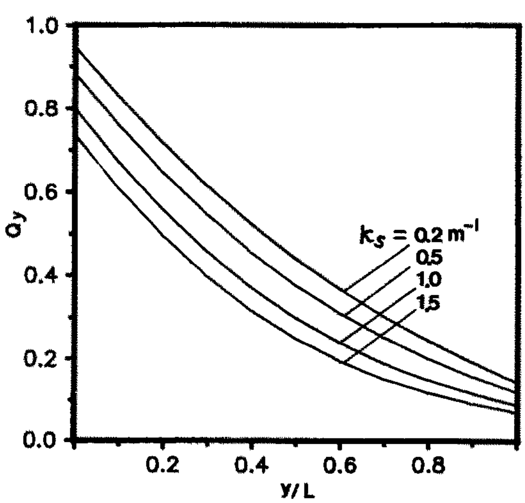

FIG. 7. Effect of scattering coefficient on the centreline net radiative heat flux in the y direction: $\varepsilon_w = 1.0$, $L = 1.0$ m, $g = 0.0$, $\kappa_a = 0.5$ m^{-1}.

3.2. *Isothermal emission problems*

The other case studied in this work is the isothermal emission problem where all the boundary walls are cold and the medium has a uniform emissive power of unity.

The effects of asymmetry factor, scattering coefficient, and absorption coefficient on the surface net radiative heat flux have been investigated by the P_1-approximation. It was found that the asymmetry factor and scattering coefficient have a negligible effect on the surface heat flux because the anisotropic effects cancel out in this symmetric radiating system. The same conclusion has been drawn by Kim and Lee [4] using the S_{14}-approximation. However, the medium absorption coefficient has a significant effect on the surface heat flux.

The P_1-approximation results of surface heat flux are compared with the exact results in Fig. 9 for different scattering albedos. The surface heat flux decreases significantly with increasing scattering albedo since an increase of scattering albedo results in a decrease of absorption coefficient. The agreement between the P_1-approximation results and the exact solutions is relatively good. The greatest errors occur near the corner region where it is believed that the radiation intensity exhibits the largest dependence on direction.

4. CONCLUSIONS

The first-order spherical harmonics approximation has been employed to solve the radiative transfer equation in an absorbing, emitting, and anisotropically scattering medium. The complex phase function was modelled by the δ-Eddington phase function approximation. It is established that the effect of scattering on radiative heat transfer can be predicted as long as the asymmetry factor of the phase function is available. It is suggested in this work to

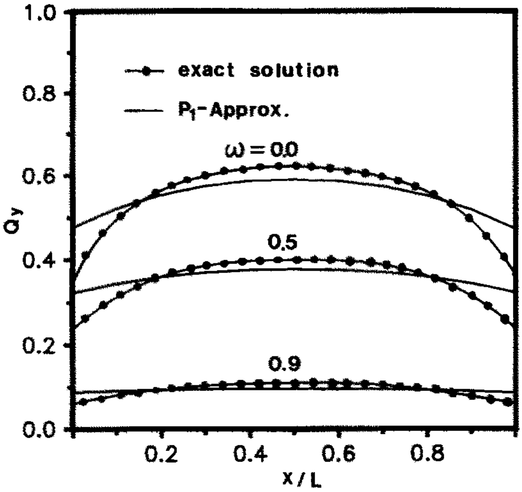

FIG. 9. Effect of scattering albedo on the net radiative heat flux on the wall surface for isothermally emitting media: $\varepsilon_w = 1.0$, $L = 1.0$ m, $g = 0.67$ (F2), $\kappa_e = 1.0$ m^{-1}.

use the concepts of the effective scattering coefficient and effective optical dimension in the study of radiative heat transfer.

Comparisons between the P_1-approximation predictions of radiative heat flux, using an improved boundary condition, with the numerically exact results of Kim and Lee [4] in a square enclosure show that the agreement is, in general, good. Although the P_1-approximation is not accurate in corner regions and in pure and highly forward scattering media under the thermal conditions considered, it should yield more accurate results in practical radiating systems since the problems studied in this work are extremely idealised.

This work also reveals the fact that effects of scattering on radiative transfer can be neglected if the medium has a highly forward scattering phase function. Based on this fact, the mixture of radiating gases and fly-ash particles in a pulverised coal-fired furnace can be treated as a non-scattering medium with confidence since fly-ash particles scatter radiation predominantly in the forward direction. However, the absorption and emission of radiation by fly-ash particles cannot be neglected.

Acknowledgement—The first author was jointly sponsored by the British Council and the Chinese government under the Sino–British Friendship Scholarship Scheme.

REFERENCES

1. M. P. Menguc and R. Viskanta, Radiative transfer in three-dimensional rectangular geometries containing inhomogeneous, anisotropic scattering media, *J. Quant. Spectrosc. Radiat. Transfer* **33**, 533–549 (1985).
2. M. P. Menguc and R. Viskanta, Radiative transfer in axisymmetric, finite cylindrical enclosures, *J. Heat Transfer* **108**, 271–276 (1986).
3. J. S. Truelove, Three-dimensional radiation in absorbing–emitting–scattering media using the discrete-ordinates approximation, *J. Quant. Spectrosc. Radiat. Transfer* **39**, 27–31 (1988).
4. T. Kim and H. Lee, Effect of anisotropic scattering on radiative heat transfer in two-dimensional rectangular enclosures, *Int. J. Heat Mass Transfer* **31**, 1711–1721 (1988).
5. M. P. Menguc and R. Viskanta, A sensitivity analysis for radiative heat transfer in a pulverised coal-fired furnace, *Combustion Sci. Technol.* **51**, 51–74 (1987).
6. F. Liu, E. S. Garbett and J. Swithenbank, On the boundary condition of the P_N-approximation used to solve the radiative transfer equation, *Int. J. Heat Mass Transfer* **35**, 2043–2052 (1992).
7. J. H. Joseph, W. J. Wiscombe and J. A. Weinman, The delta-Eddington approximation for radiative flux transfer, *J. Atmos. Sci.* **33**, 2453–2459 (1976).
8. A. L. Crosbie and G. M. Davison, Dirac-delta approximations to the scattering phase function, *J. Quant. Spectrosc. Radiat. Transfer* **33**, 391–409 (1985).
9. H. C. van de Hulst, *Multiple Light Scattering*, Chapters 10 and 14. Academic Press, New York (1980).
10. S. A. Boothroyd, A. R. Jones, K. W. Nicholson and W. Wood, Light scattering by fly ash and the applicability of Mie theory, *Combustion Flame* **69**, 235–241 (1987).
11. D. G. Goodwin and M. Mitchner, Flyash radiative properties and effects on radiative heat transfer in coal-fired systems, *Int. J. Heat Mass Transfer* **32**, 627–638 (1989).
12. F. Liu, New development in pulverised coal combustion: numerical modelling of radiative heat transfer and experimental test of a novel burner technique, Ph.D. Thesis, Department of Mechanical and Process Engineering, University of Sheffield, Sheffield, U.K., p. 145 (1990).
13. W. L. Grosshandler and S. L. P. Monteiro, Attenuation of thermal radiation by pulverised coal and char, *J. Heat Transfer* **104**, 587–593 (1982).
14. B. H. J. McKellar and M. A. Box, The scaling group of the radiative transfer equation, *J. Atmos. Sci.* **38**, 1063–1068 (1981).
15. H. Lee and R. O. Buckius, Scaling anisotropic scattering in radiation heat transfer for a planar medium, *J. Heat Transfer* **104**, 69–75 (1982).
16. T.-K. Kim and H. S. Lee, Scaled isotropic results for two-dimensional anisotropic scattering media, *J. Heat Transfer* **112**, 721–727 (1990).
17. A. C. Ratzel and J. R. Howell, Two-dimensional radiation in absorbing–emitting media using the P–N approximation, *J. Heat Transfer* **105**, 333–340 (1983).

EFFETS DE LA DIFFUSION ANISOTROPE SUR LE TRANSFERT THERMIQUE RADIATIF VUS A TRAVERS L'APPROXIMATION P_1

Résumé—La méthode d'approximation P_1, avec l'approximation de fonction de phase δ d'Eddington, est utilisée pour étudier le transfert thermique radiatif dans les milieux absorbants, émetteurs et diffusants. On établit que le facteur d'asymétrie de la fonction de phase diffusante (g) joue un rôle important dans le transfert radiatif. Les concepts de coefficient de diffusion efficace et de dimension optique efficace sont utilisées dans l'étude du transfert radiatif. Dans les milieux fortement diffusants vers l'avant ($g > 0,8$), l'effet de la diffusion peut être négligé. Dans une cavité bidimensionnelle carrée, les résultats du flux radiatif obtenus par l'approximation P_1, avec une condition aux limites améliorée, sont en bon accord avec les résultats numériques exacts de Kim et Lee (*Int. J. Heat Mass Transfer* **31**, 1711–1721 (1988)).

ANWENDUNG DER P_1-NÄHERUNG AUF DIE UNTERSUCHUNG DES EINFLUSSES ANISOTROPER STREUUNG AUF DEN STRAHLUNGSWÄRMETRANSPORT

Zusammenfassung—Zur Untersuchung des Wärmetransports in absorbierenden emittierenden und streuenden Medien wird die P_1-Näherung sowie die δ-Eddington-Phasenfunktionsnäherung verwendet. Es ist bekannt, daß der Asymmetriefaktor der Streuungsphasen-Funktion (g) den Strahlungsvorgang stark beeinflußt. Für die Untersuchung des Strahlungswärmetransports werden der effektive Streuungskoeffizient sowie die effektive optische Länge verwendet. In stark vorwärts streuenden Medien ($g > 0{,}8$) kann der Einfluß der Streuung auf den Strahlungswärmetransport vernachlässigt werden. In einem zweidimensionalen quadratischen Hohlraum wird der Strahlungswärmestrom mittels der P_1-Approximation berechnet, wobei eine verbesserte Randbedingung benutzt wird. Die Ergebnisse zeigen gute Übereinstimmung mit den numerisch exakten Berechnungen von Kim und Lee (Effect of anisotropic scattering on radiative heat transfer in twodimensional rectangular enclosures, *Int. J. Heat Mass Transfer* **31**, 1711–1721 (1988)).

ИССЛЕДОВАНИЕ ВЛИЯНИЯ АНИЗОТРОПНОГО РАССЕЯНИЯ НА РАДИАЦИОННЫЙ ТЕПЛОПЕРЕНОС С ИСПОЛЬЗОВАНИЕМ P_1-АППРОКСИМАЦИИ

Аннотация—Для исследования радиационного теплопереноса в поглощающих, излучающих и рассеивающих средах использовался метод P_1-аппроксимации в комбинации с аппроксимацией фазовой δ-функции Эддингтона. Установлено, что важную роль в радиационном переносе играет фактор асимметрии фазовой функции рассеяния (g). При исследовании радиационного теплопереноса предлагается использование понятий эффективного коэффициента рассеяния и эффективного оптического размера. В случае сильно рассеивающих вперед сред ($g > 0.8$) влиянием рассеяния на радиационный теплоперенос можно пренебречь. Результаты для радиационного теплового потока, полученные методом P_1-аппроксимации с использованием модифицированного граничного условия в случае двумерной квадратной полости, хорошо согласуются с точными численными результатами Кима и Ли (Effect of anisotropic scattering on radiative heat transfer in two-dimensional rectangular enclosures, *Int. J. Heat Mass Transfer* **31**, 1711–1721 (1988)).